Small Signal Modelling and Control of the Hydrogen Mixer for Facility E1

Final Report

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Abstract

We have undertaken the theoretical modelling of an existing liquid hydrogen (LH2) and gas hydrogen (GH2) mixer subsystem of the E1 Ground Test Facility at NASA John C. Stennis Space Center. The E1 test facility carries out comprehensive ground-based testing and certification of various liquid rocket engines and their components. The mixer described in this work is responsible for combining high pressure LH2 and GH2 to produce a hydrogen flow that meets certain thermodynamic properties before it is fed into a test article. The desired properties are maintained by precise control of the mixture of LH2 and GH2 flows. The mixer is modelled as a general multi-flow lumped volume for single constituent fluids using density and internal energy as states. The set of nonlinear differential equations is linearized about an equilibrium point and the resulting two-state, 3-input linear model is analyzed as a possible candidate for control design.

1 Introduction

The Test and Engineering Directorate at NASA John C. Stennis Space Center (SSC) is continuing its efforts to assemble a software simulation environment that captures the static and dynamic characteristics of modern and future rocket engine systems. To that end, the design of a simulation package called EASY/ROCETS has been launched that gives engineers the capability of performing dynamic fluid flow simulation and advanced control design. EASY/ROCETS is the fusion of EASY5x, a commercial package by Boeing [1], and ROCETS (ROCket Engine Transient Simulator), a rocket engine analysis package developed by United Technologies Corporation under contract to NASA [2]. The user-friendly package is foreseen to fulfill the need for an accurate and verifiable rocket engine test facility simulation environment.

We build upon the reports [3, 4, 5] and focus on one component of the high pressure hydrogen system of the E1 Test Facility, namely, the LH_2 and GH_2 mixer subsystem and associated flow controllers. The relevant components of the mixer subsystem are shown in the flow schematic of Figure 1. The LH_2 valve controls the flow of high pressure liquid H_2 from a pressurized tank; the GH_2 valve controls the flow of gaseous H_2 from a set of high pressure bottles; and the output valve controls the flow of H_2 into the test article. The primary objective of the mixer and control valves is to regulate the thermodynamic properties of the out flow of H_2 for optimal test article performance in spite of the steady depletion of the source bottles, measurement errors, modelling imperfections, and other uncertainties and perturbations.

2 Background

- Some application examples of the use of EASY/ROCETS are presented in [3] including FORTRAN listings of several modules and a brief user's manual.
- The report [4] performs a detailed derivation of the model of the RUN-TANK module as well as its implementation within EASY/ROCETS. The low pressure and high pressure LOX

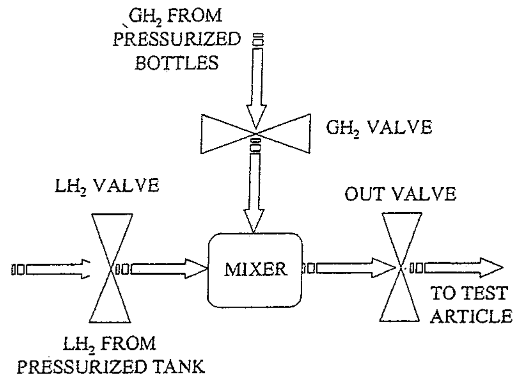

Figure 1: Diagram of HPH2 Mixer and Control Valves.

and hydrogen subsystems are simulated with EASY/ROCETS and compared against previous computer simulations with good results. In addition, various control strategies were simulated for the high pressure hydrogen subsystem. The objective being the temperature regulation of the output stream. *At present however, control designs are done in an ad-hoc manner due to the lack of a suitable mathematical model of either the tank or the mixer.*

- In [5], several upgrades and enhancements to EASY/ROCETS were reported including the inclusion of the NIST-12 thermodynamic database. Also, a PID module is introduced based on the Allen-Bradley SLC 5/03 Programmable Logic Controller (PLC). During the course of this investigation, an inconsistency was discovered in the discrete-time version of the reported PID controller equations. The corrected equations will be presented in a separate report.
- Most recently, Victor Marrero, one of the SSC 2001 Summer Undergraduate students, reported his work on validating the EASY/ROCETS models of several components of the high-pressure hydrogen system with real test data. The results are very encouraging and prompts for continued analytical studies such as the one presented in this report.

2.1 Results

- We developed an analytical model of the mixer and control valves that is suitable for dynamic analysis and control design in the vicinity of a thermodynamic equilibrium point.
- An optimal linear-quadratic regulator was designed and simulated using Matlab to illustrate the effectiveness of such a controller in compensating against perturbations that result in small deviations around the desired equilibrium. A simulation is included that indicates how one could study which valve or combination of valves is more effective in achieving a desired performance. In essence, this is a study on control authority or relegation.

- The linear model is compared with that obtained with Easy5. This is a preliminary comparison but the results are very encouraging as many of the observed discrepancies are within tolerance.

3 Mixer and Valve Models

The mixer has been modelled within EASY/ROCETS with a two-input, one output VOLM01 block, which is a general multi-flow lumped volume for single constituent fluids using density and internal energy as states [3]. In this article, we will refer to the mixer's internal energy and mass as the "state", not to be confused with the definition of state of a system as described by its thermodynamic properties. Letting $z_1(t)$, $z_2(t)$ denote the mixer's specific internal energy and mass, respectively, the two dynamic equations are given by

$$\begin{aligned} \dot{z}_1 &= \frac{1}{z_2}\left[U_{net} \; - \; W_{net}z_1 \; + \; \dot{Q}\right] && (1) \\ \dot{z}_2 &= W_{net} && (2) \end{aligned}$$

where U_{net} is the net (input minus output) energy into the mixer, W_{net} is the net (input minus output) flow, and $\dot{Q}$ is the total heat transfer rate. The net flow is simply

$$W_{net} \; = \; w_g \; + \; w_l \; - \; w_o$$

where the subscripts stand for "gas", "liquid", and "out", respectively. Since the block VOLM01 allows for flow reversals, then a total of eight possible expressions exist for the net energy U_{net} depending on the direction (sign) of the input and output flows. In this study, only positive flows are considered, that is, the gas and liquid flows are into the mixer and the output flow is out of the mixer. In essence, normal operation requires that each valve always experience a positive pressure difference across it thus disallowing flow reversal. The net input energy term is then given by

$$U_{net} \; = \; h_g w_g \; + \; h_l w_l \; - \; h_v w_o$$

where h_g, h_l, h_v denote the enthalpy of the GH_2 stream, LH_2 stream, and volume block, respectively.

The valve block within EASY/ROCETS is labelled VALV99 and it calculates the *incompressible* fluid flow value w_f given the inlet P_{IN} and outlet P_{OUT} pressures, the density of the stream ρ, and the valve flow coefficient C_{ff}, as follows: [1]

$$w_f \; = \; \sqrt{\rho \, \rho_w (P_{IN} - P_{OUT})} \; C_{ff} \; = \; \alpha_f C_{ff} \qquad (3)$$

where ρ_w is the density of water, and we have assumed that $P_{IN} > P_{OUT}$. Substituting the expressions for the valve flows and the net internal energy and net flow into equations (1)-(2), and

[1] It is with very high probability that in order to achieve a closer match with physical reality, some of the valve blocks will have to be modified or replaced by a valve model that simulates *compressible* flow [6]. That study is left for future research.

treating the valve coefficients as inputs, the dynamic model of the mixer is a 2-state, 3-input system of nonlinear differential equations of the form

$$\dot{z}_1 = F_1(z_1,\ z_2,\ C_{fg},\ C_{fl},\ C_{fo}) \tag{4}$$

$$\dot{z}_2 = F_2(z_1,\ z_2,\ C_{fg},\ C_{fl},\ C_{fo}) \tag{5}$$

where $F_1(\cdot)$ and $F_2(\cdot)$ are nonlinear functions of the state and valve coefficients.

For the remainder of the article, we will denote constant or equilibrium values of any variable by an upper bar $\bar{(\cdot)}$. Given constant values of valve flow coefficients $\bar{C}_f = [\bar{C}_{fg}\ \bar{C}_{fl}\ \bar{C}_{fo}]^T$ (superscript T denotes transposition) and constant fluid properties, the state of the model

$$z(t) = \begin{bmatrix} z_1: & \text{Internal Energy} \\ z_2: & \text{Mass} \end{bmatrix}$$

reaches a constant equilibrium point $\bar{z}$. Next, consider perturbing such an equilibrium by small signals $x(t)$ and $u(t)$ so that

$$z(t) = \bar{z} + x(t) \qquad \text{and} \qquad C_f(t) = \bar{C}_f + u(t)$$

where $u(t)$ denotes a small valve coefficient correction signal. Then, a standard linearization of equations (4)-(5) results in the small signal model

$$\dot{x} = Ax + Bu \tag{6}$$

where $x(t)$ is the small perturbation state vector, $u(t)$ is the small perturbation control signal, and the two-by-two matrix A and two-by-three matrix B are given by

$$A = \begin{bmatrix} \frac{\partial F_1}{\partial z_1} & \frac{\partial F_1}{\partial z_2} \\ \frac{\partial F_2}{\partial z_1} & \frac{\partial F_2}{\partial z_2} \end{bmatrix}; \qquad B = \begin{bmatrix} \frac{\partial F_1}{\partial C_{fg}} & \frac{\partial F_1}{\partial C_{fl}} & \frac{\partial F_1}{\partial C_{fo}} \\ \frac{\partial F_2}{\partial C_{fg}} & \frac{\partial F_2}{\partial C_{fl}} & \frac{\partial F_2}{\partial C_{fo}} \end{bmatrix}$$

where the partial derivatives are evaluated at the equilibrium state $\bar{z}$ and constant valve flow coefficient vector $\bar{C}_f$.

The model (6) describes the dynamics of the mixer model in the vicinity of the equilibrium under consideration. Ideally, both $x(t)$ and $u(t)$ are zero; in practice, the state $z(t)$ deviates from the desired $\bar{z}$ thus necessitating a corrective action by $u(t)$. The model is suitable for a variety of analysis studies such as stability, controllability, and regulation/tracking of the equilibrium. The stability study is relevant since it reveals an important characteristic of the equilibrium $\bar{z}$ under no control, that is, with $u(t) = 0$. Appropriate action must be taken by the control $u(t)$ in the event that $\bar{z}$ be unstable (worst case) or even "poorly behaved". The notion of controllability reveals whether a suitable control $u(t)$ exists such that the perturbation state $x(t)$ may be steered to zero. Moreover, it is possible to study which valve or combination of valves is most effective. Finally, the equilibrium regulation/tracking study refers to the actual control design to regulate $x(t)$ to zero or to track a desired equilibrium trajectory as a function of time.

Typically, in the control literature an output equation of the form

$$y = Cx + Du$$

where matrices C and D are appropriately dimensioned is appended to the model (6) to account for the measurement of certain variables for example, temperature, pressure, or flow. The *state postulate* [2] implies that the state $z(t) = \bar{z} + x$ is directly available for control design if at least two independent thermodynamic properties were measured, for example, temperature and pressure. Therefore, it is reasonable to assume at least initially that

$$y = x = z - \bar{z} \tag{7}$$

3.1 Calculation of Equilibrium Points

By definition, an equilibrium point $\bar{z}$ of equations (4)-(5) satisfies

$$\dot{\bar{z}}_1 = 0 \implies \bar{h}_g \bar{w}_g + \bar{h}_l \bar{w}_l - \bar{h}_v \bar{w}_o + \dot{\bar{Q}} = 0 \tag{8}$$

$$\dot{\bar{z}}_2 = 0 \implies \bar{w}_g + \bar{w}_l - \bar{w}_o = 0 \tag{9}$$

Given a set of required output mass flow, pressure, and temperature $\bar{w}_o$, $\bar{P}_o$, $\bar{T}_o$, then the property routine MIPROPS was used to determine the enthalpy $\bar{h}_o = \bar{h}_v$. [3] Next, select an appropriate set of GH_2 and LH_2 properties. Then, using the equilibrium conditions (8)-(9) with $\dot{\bar{Q}} = 0$, and the flow expressions (3) for each valve, the following two linear equations in the gas and liquid flow coefficients are obtained

$$\begin{bmatrix} \bar{\alpha}_g & \bar{\alpha}_l \\ \frac{\bar{h}_g \bar{\alpha}_g}{\bar{h}_v} & \frac{\bar{h}_l \bar{\alpha}_l}{\bar{h}_v} \end{bmatrix} \begin{bmatrix} \bar{C}_{fg} \\ \bar{C}_{fl} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \bar{w}_o$$

which admit the unique solution

$$\bar{C}_{fg} = \frac{\bar{h}_v - \bar{h}_l}{\bar{h}_g - \bar{h}_l} \frac{1}{\bar{\alpha}_g} \bar{w}_o \implies \bar{w}_g = \frac{\bar{h}_v - \bar{h}_l}{\bar{h}_g - \bar{h}_l} \bar{w}_o \tag{10}$$

$$\bar{C}_{fl} = \frac{\bar{h}_g - \bar{h}_v}{\bar{h}_g - \bar{h}_l} \frac{1}{\bar{\alpha}_l} \bar{w}_o \implies \bar{w}_l = \frac{\bar{h}_g - \bar{h}_v}{\bar{h}_g - \bar{h}_l} \bar{w}_o \tag{11}$$

The mixer equilibrium state $\bar{z}_2$ is simply

$$\bar{z}_2 = V \bar{\rho}_v$$

and $\bar{z}_1$ is determined from a suitable thermodynamic property table or from MIPROPS.

[2] For a simple compressible substance, any thermodynamic property is at most a function of two other independent thermodynamic properties.

[3] Enthalpy across each valve remains constant by conservation of energy.

3.2 Calculation of Matrices A and B

To determine the linear perturbation model (6), it is necessary to find the analytic expressions of the indicated partial derivatives. For simplicity we include only those terms that do not evaluate to zero at the equilibrium point. Omitting the details, these are found to be:

$$\left[\frac{\partial F_1}{\partial z_1}\right]_{eq} = \frac{1}{\bar{z}_2}\left[(h_g - z_1)\,C_{fg}\frac{\partial \alpha_g}{\partial z_1} + (h_l - z_1)\,C_{fl}\frac{\partial \alpha_l}{\partial z_1} - (h_v - z_1)\,C_{fo}\frac{\partial \alpha_o}{\partial z_1} - w_o\frac{\partial h_v}{\partial z_1}\right]_{eq}$$

$$\left[\frac{\partial F_1}{\partial z_2}\right]_{eq} = \frac{1}{\bar{z}_2}\left[(h_g - z_1)\,C_{fg}\frac{\partial \alpha_g}{\partial z_2} + (h_l - z_1)\,C_{fl}\frac{\partial \alpha_l}{\partial z_2} - (h_v - z_1)\,C_{fo}\frac{\partial \alpha_o}{\partial z_2} - w_o\frac{\partial h_v}{\partial z_2}\right]_{eq}$$

$$\left[\frac{\partial F_2}{\partial z_1}\right]_{eq} = \left[C_{fg}\frac{\partial \alpha_g}{\partial z_1} + C_{fl}\frac{\partial \alpha_l}{\partial z_1} - C_{fo}\frac{\partial \alpha_o}{\partial z_1}\right]_{eq}$$

$$\left[\frac{\partial F_2}{\partial z_2}\right]_{eq} = \left[C_{fg}\frac{\partial \alpha_g}{\partial z_2} + C_{fl}\frac{\partial \alpha_l}{\partial z_2} - C_{fo}\frac{\partial \alpha_o}{\partial z_2}\right]_{eq}$$

$$\left[\frac{\partial F_1}{\partial C_{fg}}\right]_{eq} = \frac{\bar{\alpha}_g}{\bar{z}_2}\left(\bar{h}_g - \bar{z}_1\right)\ ; \qquad \left[\frac{\partial F_1}{\partial C_{fl}}\right]_{eq} = \frac{\bar{\alpha}_l}{\bar{z}_2}\left(\bar{h}_l - \bar{z}_1\right)$$

$$\left[\frac{\partial F_1}{\partial C_{fo}}\right]_{eq} = -\frac{\bar{\alpha}_o}{\bar{z}_2}\left(\bar{h}_v - \bar{z}_1\right)\ ; \qquad \left[\frac{\partial F_2}{\partial C_{fg}}\right]_{eq} = \bar{\alpha}_g$$

$$\left[\frac{\partial F_2}{\partial C_{fl}}\right]_{eq} = \bar{\alpha}_l\ ; \qquad \left[\frac{\partial F_2}{\partial C_{fo}}\right]_{eq} = -\bar{\alpha}_o$$

where

$$\left[\frac{\partial \alpha_g}{\partial z_1}\right]_{eq} = \frac{-1}{2}\left[\frac{\rho_g\rho_w}{\alpha_g}\frac{\partial P_v}{\partial z_1}\right]_{eq}\ ; \qquad \left[\frac{\partial \alpha_l}{\partial z_1}\right]_{eq} = \frac{-1}{2}\left[\frac{\rho_l\rho_w}{\alpha_l}\frac{\partial P_v}{\partial z_1}\right]_{eq}$$

$$\left[\frac{\partial \alpha_o}{\partial z_1}\right]_{eq} = \frac{1}{2}\left[\frac{\rho_v\rho_w}{\alpha_o}\frac{\partial P_v}{\partial z_1}\right]_{eq}\ ; \qquad \left[\frac{\partial \alpha_g}{\partial z_2}\right]_{eq} = \frac{-1}{2}\left[\frac{\rho_g\rho_w}{\alpha_g}\frac{\partial P_v}{\partial z_2}\right]_{eq}$$

$$\left[\frac{\partial \alpha_l}{\partial z_2}\right]_{eq} = \frac{-1}{2}\left[\frac{\rho_l\rho_w}{\alpha_l}\frac{\partial P_v}{\partial z_2}\right]_{eq}\ ; \qquad \left[\frac{\partial \alpha_o}{\partial z_2}\right]_{eq} = \frac{1}{2}\left[\frac{\rho_w}{V\alpha_o}\left(P_v - P_o + z_2\frac{\partial P_v}{\partial z_2}\right)\right]_{eq}$$

Note that the evaluation of $\frac{\partial h_v}{\partial z}$ and $\frac{\partial P_v}{\partial z}$ would have to be done numerically within the EASY/ROCETS environment based on its property routines. Alternatively, we used the property software package called `MIPROPS` that asks the user to enter values for pressure, density, and temperature, and returns all the properties. A nominal property set corresponding to a mixer pressure of $P_v = 6000\ psia$ and density $\rho_v = 2.914\ lb/ft^3$ was chosen. By varying the pressure while keeping the density constant, we generated property data as shown in Table 1. The top plots of Figure 2 and the indicated straight-line fits provide the relation between enthalpy h_v and pressure P_v to the internal energy z_1 while keeping the mass z_2 (or equivalently the density) constant. The temperature that the program converges to is seen to deviate slightly from the entered nominal value of $T = -200$ F; these deviations are likely to be caused by numerical convergence within the program itself.

Similarly, data relating enthalpy h_v and pressure P_v to the mass z_2 while keeping the internal energy z_1 constant was obtained by entering a density value at a nominal temperature and adjusting the pressure until the returned value for the internal energy was $z_1 = 407.4$ BTU/lbm. This was repeated for a range of density values. Representative data is listed in Table 2 and the linear-fit relations are given in the bottom plot if Figure 2.

Table 3 lists the pertinent equilibrium data corresponding to a given set of outflow requirements for a test-article. A second set of data is given in Table 4 corresponding to a new equilibrium that results when the properties of the gas and liquid in-streams change slightly. Note that the valve coefficients are not modified; consequently, the outflow properties deviate from the desired values.

Table 5 lists nine data points of $(P,\ T,\ h)$ as a function of the independent variables $(z_1,\ z_2)$ for the *fixed* gas-, liquid-, and out-stream properties, and *fixed* valve positions given in Table 3. The equilibrium conditions (8) and (9) are computed and plotted in Figure 3 illustrating the fact that for a given set of in- and out-stream properties and fixed valve positions, the resulting equilibrium point is *isolated.* That is, no other equilibrium points exist within an infinitesimal neighborhood around that equilibrium point. We infer that is the case for any other equilibrium point.

4 Control Design

A preliminary control design has been completed and it is detailed in this section. Consider the linear model (6) valid in the neighborhood of the equilibrium listed in Table 3. The matrices A and B are evaluated to be

$$A = \begin{bmatrix} -45.709 & -7686.3 \\ -0.63844 & -155.54 \end{bmatrix}; \qquad B = \begin{bmatrix} 609.65 & -64.95 & -33.99 \\ 2.6001 & 2.1211 & -0.6488 \end{bmatrix}$$

In classical control design one could proceed to work with the transfer functions associated with this system. Consider the two states as outputs, and since there are three inputs, then there are six transfer functions given by

$$\begin{aligned} \frac{X_1(s)}{U_g(s)} &= \frac{609.65(s+122.8)}{(s+189.6)(s+11.61)}; & \frac{X_1(s)}{U_l(s)} &= \frac{-64.95(s+406.6)}{(s+189.6)(s+11.61)} \\ \frac{X_1(s)}{U_o(s)} &= \frac{-33.99(s+8.86)}{(s+189.6)(s+11.61)}; & \frac{X_2(s)}{U_g(s)} &= \frac{2.6(s-104)}{(s+189.6)(s+11.61)} \\ \frac{X_2(s)}{U_l(s)} &= \frac{2.12(s+65.26)}{(s+189.6)(s+11.61)}; & \frac{X_2(s)}{U_o(s)} &= \frac{-0.65(s+12.25)}{(s+189.6)(s+11.61)} \end{aligned}$$

It is in general difficult to design PID-style controllers for each loop because of the interactions between channels. The approach chosen in this work is based on the state-space model (6) rather than on the individual transfer functions.

Given the system (6), a performance criterion of the form

$$J = \int_0^\infty \left(x^T(t)Qx(t) + u^T(t)Ru(t) \right) dt$$

is minimized over all possible control signals $u(t)$. For technical reasons, the matrix Q must be positive-semidefinite and penalizes how far the state $x(t)$ deviates from the origin. Similarly, the matrix R must be positive-definite and penalizes how much control effort is used to drive the state $x(t)$ to the origin. The resulting optimal controller is given by the linear full-state feedback law

$$u(t) = -Kx(t) \tag{12}$$

where the matrix K is constant and is found by solving the steady-state Riccati Equation

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \qquad \Longrightarrow K = R^{-1}B^T P$$

The closed-loop system

$$\dot{x} = [A - BK]x$$

is guaranteed to be asymptotically stable, that is, $x(t) \to 0,\ t \to \infty$.

As a simulation example, select the following weighting matrices:

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; \qquad R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

so that it is equally important to steer x_1 or x_2 to zero, and the three valves have equal authority in achieving the stated goal. The optimal gain matrix is found to be

$$K = \begin{bmatrix} 0.90578 & -9.35771 \\ -0.14018 & 2.613 \\ -0.041332 & 0.182310 \end{bmatrix}$$

The resulting control policy and state trajectories are shown in Figure 4 and Figure 5, respectively. Note that the gas valve shows to have the biggest influence. As a comparison, change the control weighting matrix R to

$$R = \begin{bmatrix} 20 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

so that the gas valve controller is *penalized* twenty times more than the liquid or out valves. The optimal gain matrix is found to be

$$K = \begin{bmatrix} 0.12697 & -2.8667 \\ -0.54362 & 17.278 \\ -0.084231 & 0.85055 \end{bmatrix}$$

and the resulting control and state histories are given in in Figure 6 and Figure 7, respectively. Observe that indeed the authority of the gas valve has been reduced with a corresponding increase in the efforts of the liquid and out valves. Compared to the previous case, the perturbation states are seen to take slightly longer to reach the origin.

5 Conclusions and Further Research

An analytical small-signal model of the high-pressure H_2 mixer and control valves has been derived. The model is suitable for a variety of analysis and control designs valid in the vicinity of an operating or equilibrium point. A classical approach to control design would involve the consideration of a total of six single-input, single-output transfer functions. Unless the control loops are weakly coupled, such a design is difficult to complete because of the interaction among the control channels. Instead, a state-variable control design has been pursued that results in an optimal full-state constant feedback law that can be implemented in real-time. As illustrated in the simulations, it is straightforward to design the regulator and it is possible to assign different authorities to each control valve.

Topics for further research include:

- Introduce a valve model for compressible flows as well as valve dynamics, such as friction and possibly other nonlinear effects. It has been experimentally found that there is a "dip" in the "Flow" versus "Valve Position" curve that may result in unstable operation [7]. A heuristic approach has been used to solve this problem.

- Investigate the problem of equilibrium trajectory tracking. This refers to the design of control policies that steer the PVT-curve as a function of time.

- Investigate the combined tank/mixer control problem. This is a six-state and four control-valve problem. Clearly, the quality of the out-flow properties is affected by the precise regulation of both the tank and mixer internal dynamic states.

- Implement these findings within EASY/ROCETS. The following represents a modest effort along these lines. The isolated mixer has been built in EASY/ROCETS as shown in Figure 8. The "Linear Analysis" module was used to identify the resulting equilibrium point yielding $\rho_v = 1.68648E{-}03\ lbm/in^3$ and $E_v = 407.32\ BTU/lbm$. These compare very well with $\bar{z}_2/V = 1.685E-03\ lbm/in^3$ and $\bar{z}_1 = 407.4\ BTU/lbm$, respectively. The same module generates the following linearized system model (the subscript ER denotes the EASY/ROCETS matrices):

$$A_{ER} = \begin{bmatrix} -60.34 & -7634.57 \\ -0.919 & -155.7 \end{bmatrix}; \qquad B_{ER} = \begin{bmatrix} 608.69 & -64.82 & -33.97 \\ 2.59 & 2.12 & -0.649 \end{bmatrix}$$

  which are to be compared with the analytical matrices

$$A = \begin{bmatrix} -45.709 & -7686.3 \\ -0.63844 & -155.54 \end{bmatrix}; \qquad B = \begin{bmatrix} 609.65 & -64.95 & -33.99 \\ 2.6001 & 2.1211 & -0.6488 \end{bmatrix}$$

  The cause for the discrepancy in the first column of matrices A_{ER} and A should be investigated further. The rest of the entries are deemed to be within acceptable tolerances. Finally, the system's natural frequencies are the eigenvalues of the system matrix A_{ER}. These are easily found to be $-11.39,\ -203.7$. The eigenvalues of A are $-11.61,\ -189.6$. The natural or unforced responses corresponding to these sets of eigenvalues (poles) are quite similar.

- *Last but definitely not least, we must validate these findings with real data.*

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P (psia)	T (F)	z1 (BTU/lb)	h (BTU/lb)
5980	-200.7	405.2	785.2
5985	-200.51	405.8	786.1
5990	-200.33	406.3	787
5995	-200.15	406.9	787.8
6000	-199.96	407.4	788.7
6005	-199.78	408	789.6
6010	-199.6	408.5	790.5
6015	-199.41	409.1	791.3
6020	-199.23	409.6	792.2

Table 1: Hydrogen Properties at Constant Density $\rho = 2.914\ lb/ft^3$.

P (psia)	T (F)	h (BTU/lb)	ρ (lb/ft^3)
5595	-200.88	777.4	2.8
5770	-200.47	782.3	2.85
5949	-200.08	787.3	2.9
5985	-200.01	788.2	2.91
6000	-199.96	788.7	2.914
6132	-199.7	792.3	2.95
6320	-199.32	797.5	3
6513	-198.94	802.8	3.05

Table 2: Hydrogen Properties at Constant Internal Energy $z_1 = 407.4\ BTU/lb$.

	$\bar{P}$ (psi)	$\bar{T}$ (F)	$\bar{\rho}$ (lbm/ft^3)	$\bar{h}$ (BTU/lbm)	C_{vopen}	$\%O\bar{P}EN$	$\bar{w}$ lbm/s	$\bar{z}_1$ BTU/lbm	$\bar{z}_2$ lbm
GH_2 IN	13500	90	2.91	2113.6	230	4.35	-	-	-
GH_2 OUT	6000	135	1.53	2113.6	-	-	26.0	-	-
LH_2 IN	8500	-400	5.81	184.6	115	23.4	-	-	-
LH_2 OUT	6000	-370	5.02	184.6	-	-	57.0	-	-
Mixer	6000	-200	2.91	788.7	-	-	-	407.4	7.28
Outflow	5533	-198	2.76	788.7	270	47.37	83.0	-	-

Table 3: Equilibrium Data. Underlined Values are Computed. (IN=Into Valve; OUT=Out of Valve)

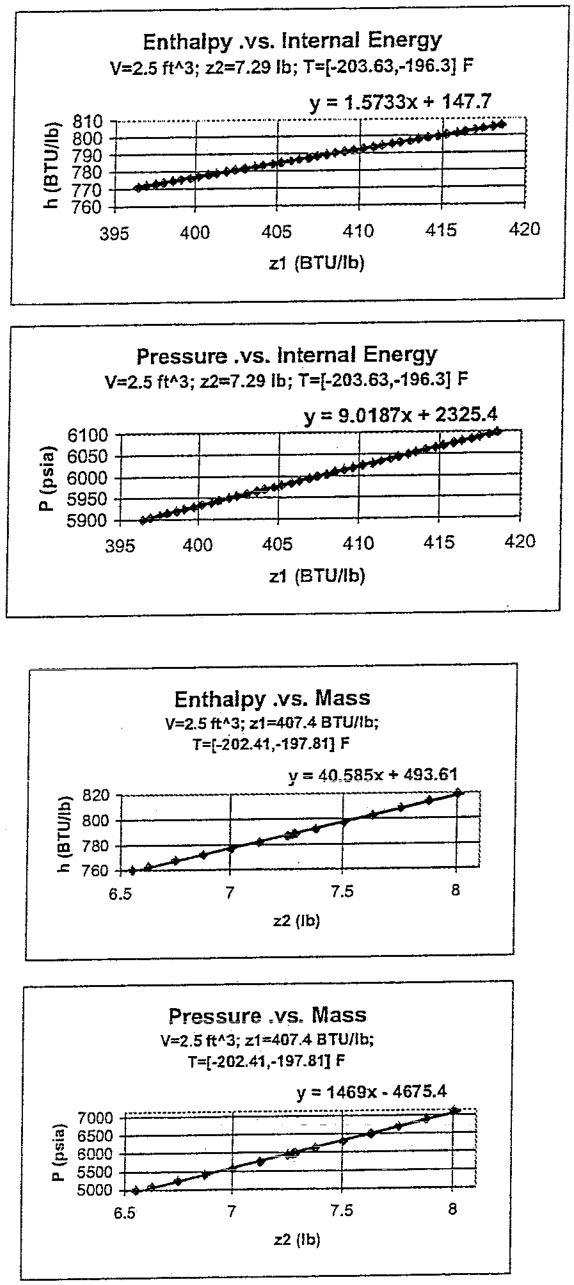

Figure 2: Top: H_2 Enthalpy and Pressure Variation with Internal Energy at Constant Density. Bottom: H_2 Enthalpy and Pressure Variation with Mass at Constant Internal Energy.

	P (psi)	T (F)	ρ (lbm/ft^3)	h (BTU/lbm)	$C_{v open}$	$\%OPEN$	w lbm/s	x_1 BTU/lbm	x_2 lbm
GH_2 IN	13400	91	2.891	2115.1	230	<u>4.35</u>	-	-	-
GH_2 OUT	5948	135.8	1.52	2115.1	-	-	<u>25.84</u>	-	-
LH_2 IN	8287	-398	5.76	182.6	115	<u>23.4</u>	-	-	-
LH_2 OUT	5948	-370.2	5.0	182.6	-	-	<u>54.97</u>	-	-
Mixer	5948	-197	2.87	800.6	-	-	-	<u>9.9</u>	<u>-0.09</u>
Outflow	5500	-195	2.73	800.6	270	<u>47.37</u>	<u>80.81</u>	-	-

Table 4: Perturbed Equilibrium Data. Underlined Values are Computed. (IN=Into Valve; OUT=Out of Valve)

P (psi)	T (F)	h (BTU/lbm)	$z1$ (BTU/lbm)	z_2 (lbm)
5980	-140	1042.9	600	6.25
5985	-156	974.5	548.2	6.5
5990	-171	910.7	499.9	6.75
5995	-185	851.4	455	7
6000	-200	788.7	407.4	7.3
6005	-210	744.9	374.2	7.5
6010	-222	697.1	338.1	7.75
6015	-232	652.6	304.6	8
6020	-242	611.3	273.5	8.25

Table 5: Data Used to Compute the Equilibrium Conditions EQ1 (8) and EQ2 (9).

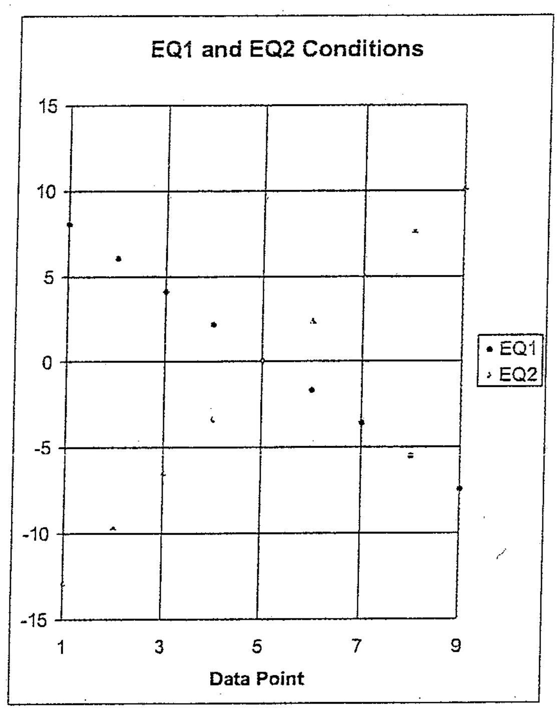

Figure 3: Equilibrium Conditions Illustrating that the Equilibrium Point (Data Point 5) is Isolated.

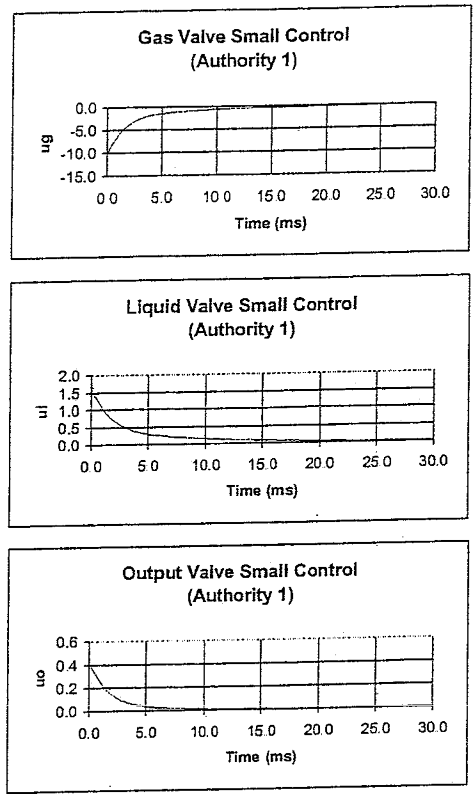

Figure 4: Optimal Full-State Feedback Controllers for Case 1.

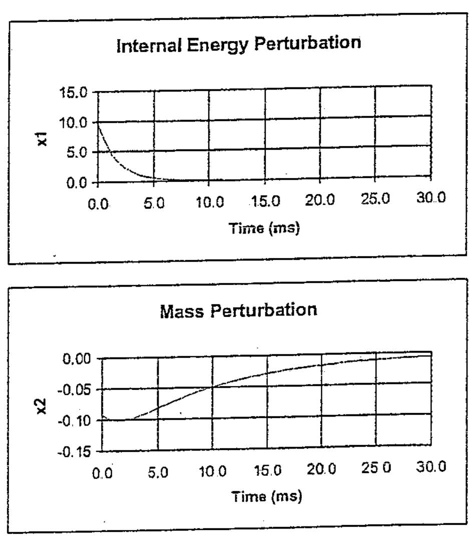

Figure 5: Optimal State Trajectories for Case 1.

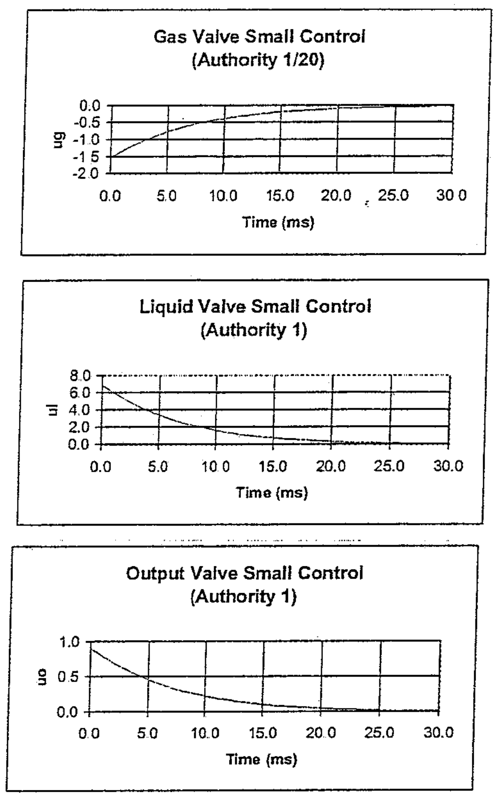

Figure 6: Optimal Full-State Feedback Controllers for Case 2.

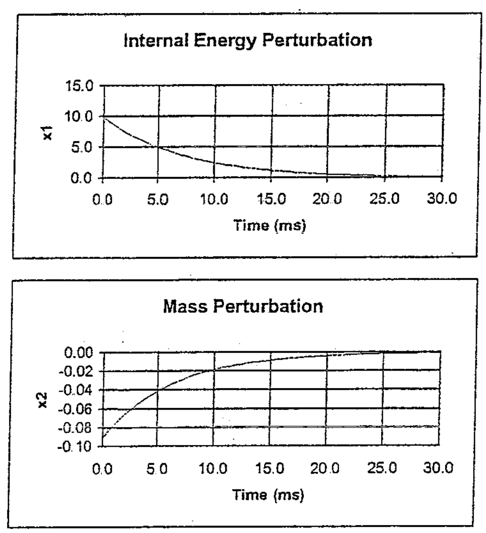

Figure 7: Optimal State Trajectories for Case 2.

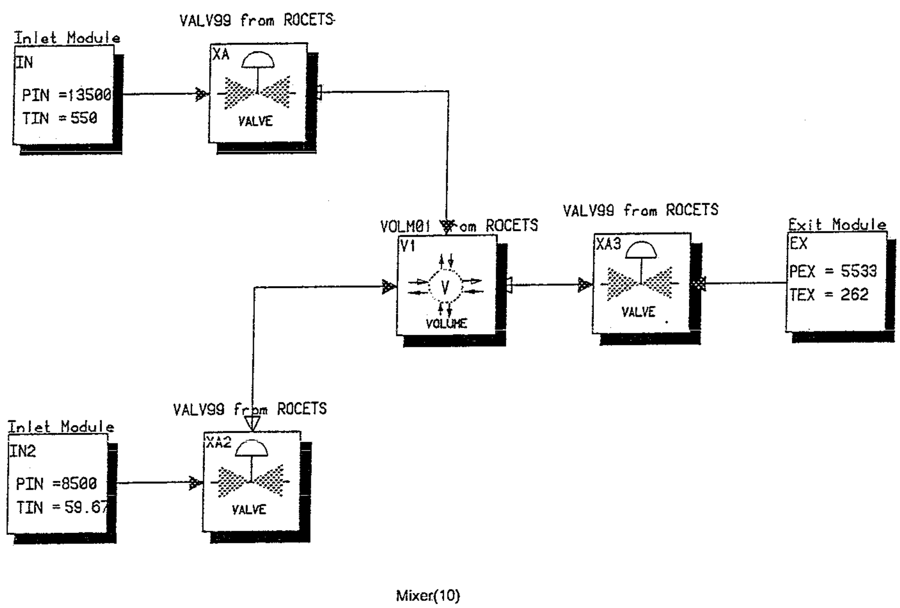

Figure 8: EASY/ROCETS Implementation of the Mixer and Control Valves.

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